TOTAL MARKS

Please paste the barcoded label here

MATHEMATICS: PAPER II

EXAMINATION NUMBER

Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.

2. Read the questions carefully.

3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**

4. Diagrams are not necessarily drawn to scale.

5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6. Ensure that your calculator is in **DEGREE** mode.

7. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**

8. Round off to **ONE DECIMAL PLACE** unless otherwise stated.

9. It is in your own interest to write legibly and to present your work neatly.

10. TWO blank pages (pages 27 and 28) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the number of your answer should you use this extra space.
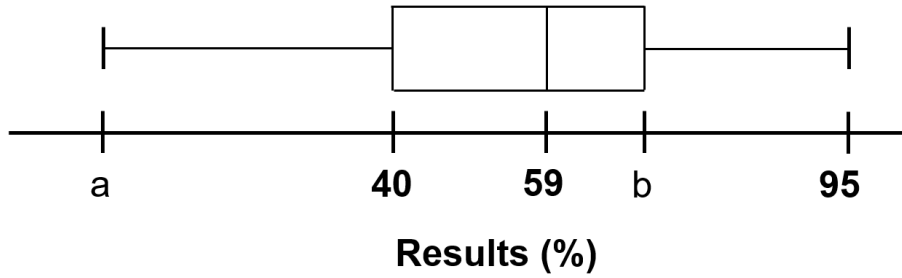
FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	TOTAL
9	22	12	5	11	14	7	7	17	8	17	13	8	/150

SECTION A

QUESTION 1

The percentage results obtained by 26 learners in a mathematics test is displayed in the box and whisker plot below:

Results (%)

If the range of the data is **80** and the interquartile range (IQR) is **30**:

(a)   Determine the value of:

(1)   a

(1)

(2)   b

(1)

(b)   Determine whether the minimum result obtained (using your answer to (a)(1)) is an outlier or not.

Use the formula $Q_1 - 1.5 \times \text{IQR}$

(2)

(c)   The table illustrates the approximate time spent (x) in minutes by 7 learners studying for a mathematics test and their mark obtained as a percentage.

Time spent (x) in minutes	0	90	90	80	90	120	150
Mark obtained (y) as a %	15	59	60	73	85	90	95

(1)   Predict, using the equation of the least squares regression line in the form $y = a + bx$ for this data, what mark a learner who studies for 180 minutes will obtain.
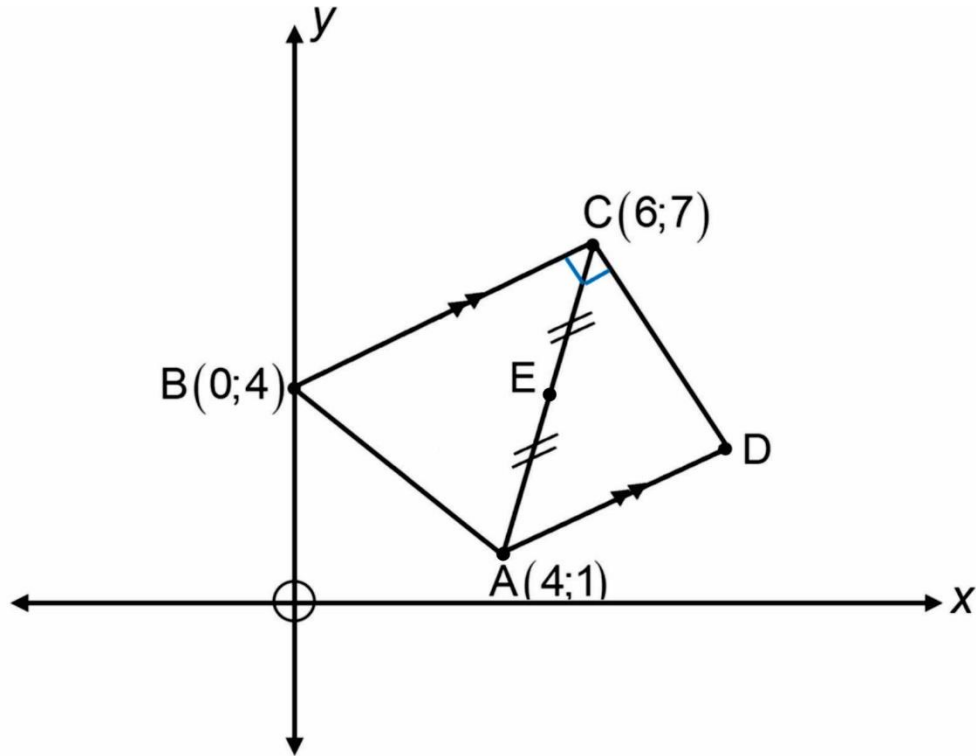**Round your answers to 3 decimal places.**

(3)

(2)   Is this prediction in (c)(1) a reliable one? Explain.

(2)
[9]

QUESTION 2

In the diagram:

- ABCD is a trapezium with AD ∥ BC and $B\hat{C}D = 90°$.
- Points A(4;1), B(0;4) and C(6;7) are given.
- E is a point on CA so that EC = EA.

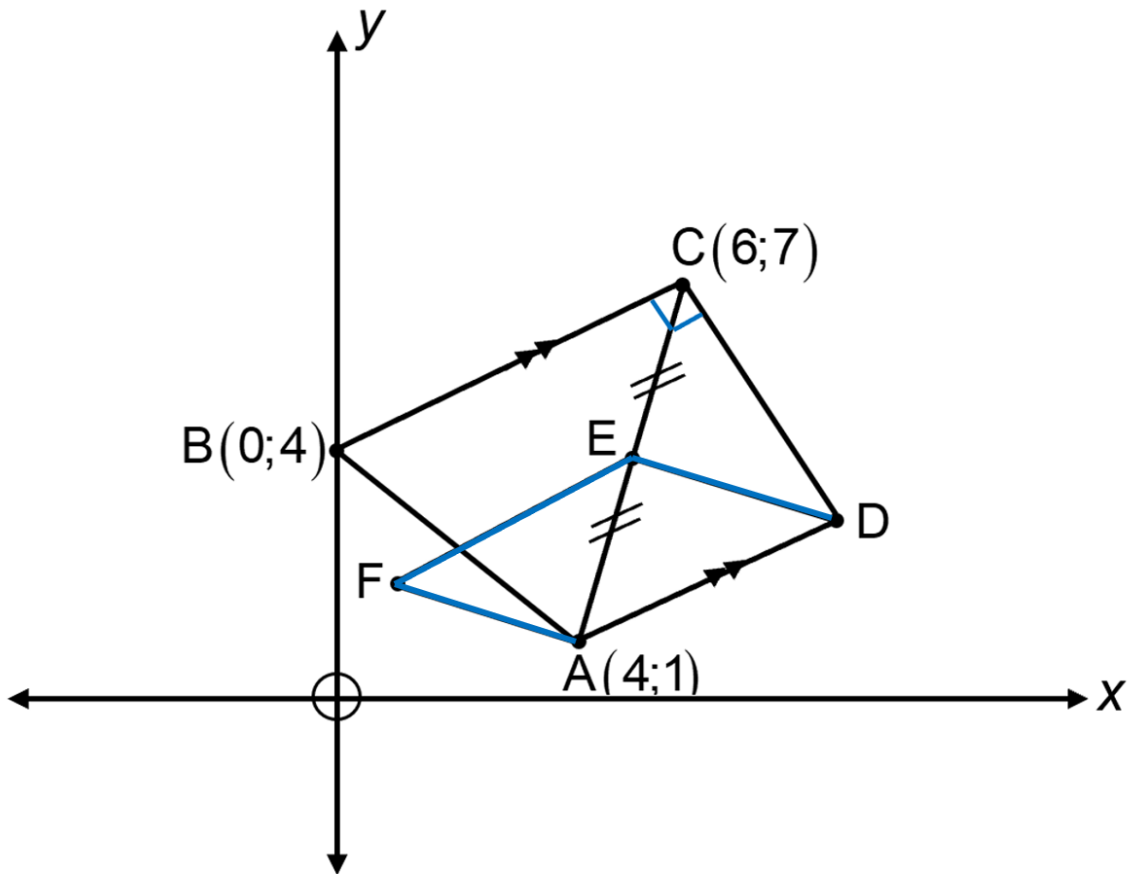(a)   (1)   Determine the gradient of BC.

(2)

       (2)   Determine the equation of the line CD.

(3)

(b)   Determine the equation of AD and hence the coordinates of D.

(5)

(c)   It is further given that point F is a point in the first quadrant so that
        FEDA is a parallelogram. The coordinates of D(8;3) are also given now.

(1)   Determine the coordinates of F.

(3)

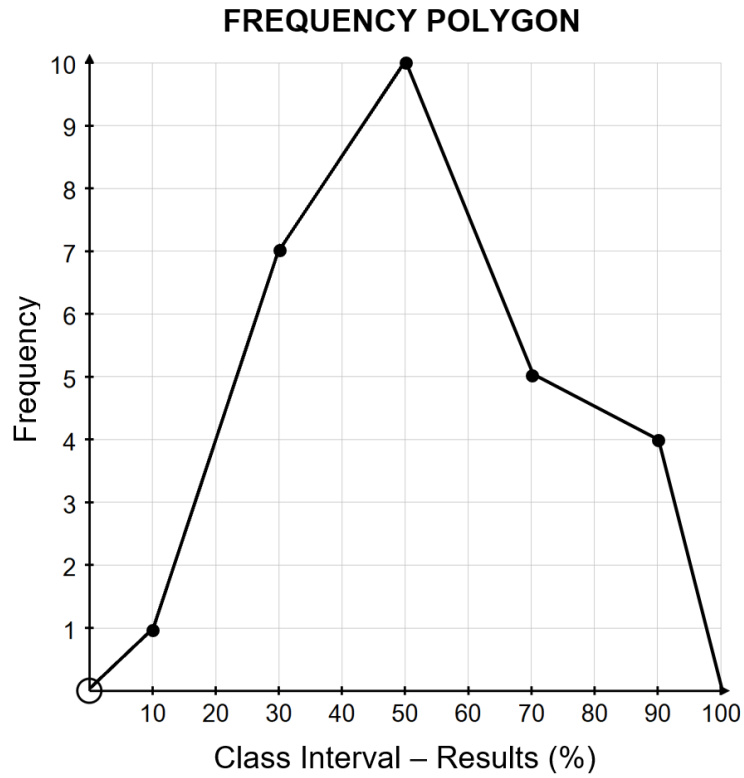(2)   Determine $E\hat{A}D$, correct to one decimal place.

(4)

(3)   Hence, or otherwise determine the area of parallelogram ADEF.

(5)
[22]

QUESTION 3

The frequency polygon represents the results obtained by 27 learners in a mathematics test.

The class intervals are given as $[0;20)$; $[20;40)$; $[40;60)$; $[60;80)$; $[80;100]$.

FREQUENCY POLYGON

(a)   Determine the estimated mean of this set of results.

(3)

(b)   What percentage of learners obtained a result in the interval $60 \leq x < 80$?

(1)

(c)   Use the information given on the frequency polygon to draw a Cumulative Frequency
        Curve (Ogive) that represents the results of this group of learners.

(3)

(d)   (1)   Indicate on the horizontal axis of your cumulative frequency curve
                in (c) where you would read off:

                (i)   the median

(1)

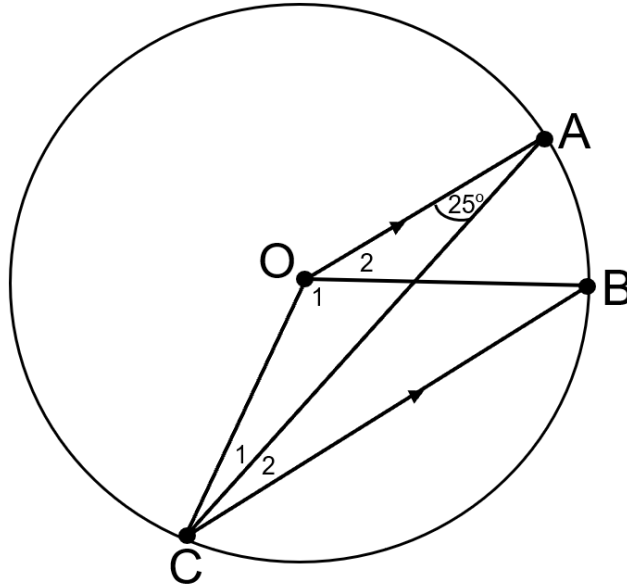   (ii)   the values required to determine the IQR.

(2)

        (2)   Using your answer to (a) and (d)(1)(i), describe the skewness of the data set.
                Explain.

(2)
[12]

QUESTION 4

In the diagram:

- The points A, B and C all lie on the circle with centre O.
- Line OA ∥ CB.

- $O\hat{A}C = 25°$
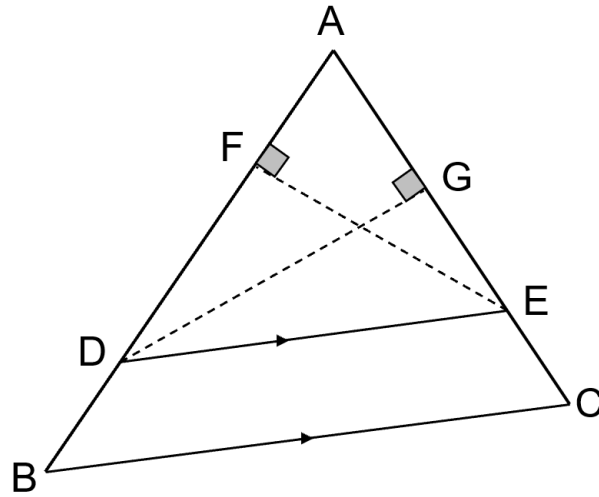

(a) Determine: $C\hat{O}A$

(2)

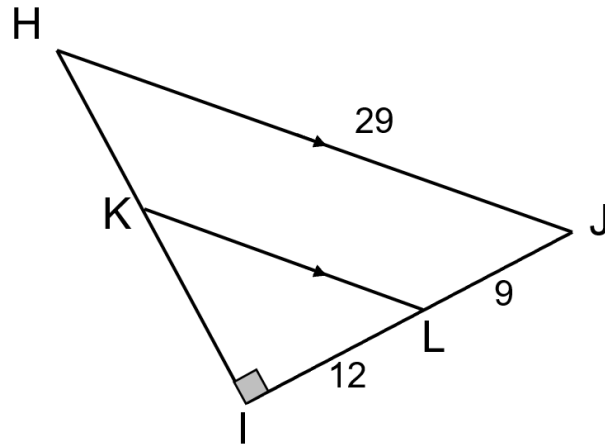(b) Determine: $\hat{O}_1$

(3)
[5]

QUESTION 5

(a)   Use the diagram below to prove the theorem that states that **a line drawn parallel to one side of a triangle divides the other two sides proportionally**.

Required to prove: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Proof:

(6)

(b)   In the diagram:
- HIJ is a right-angled triangle at I.
- L lies on IJ such that $JL = 9$ units and $LI = 12$ units.
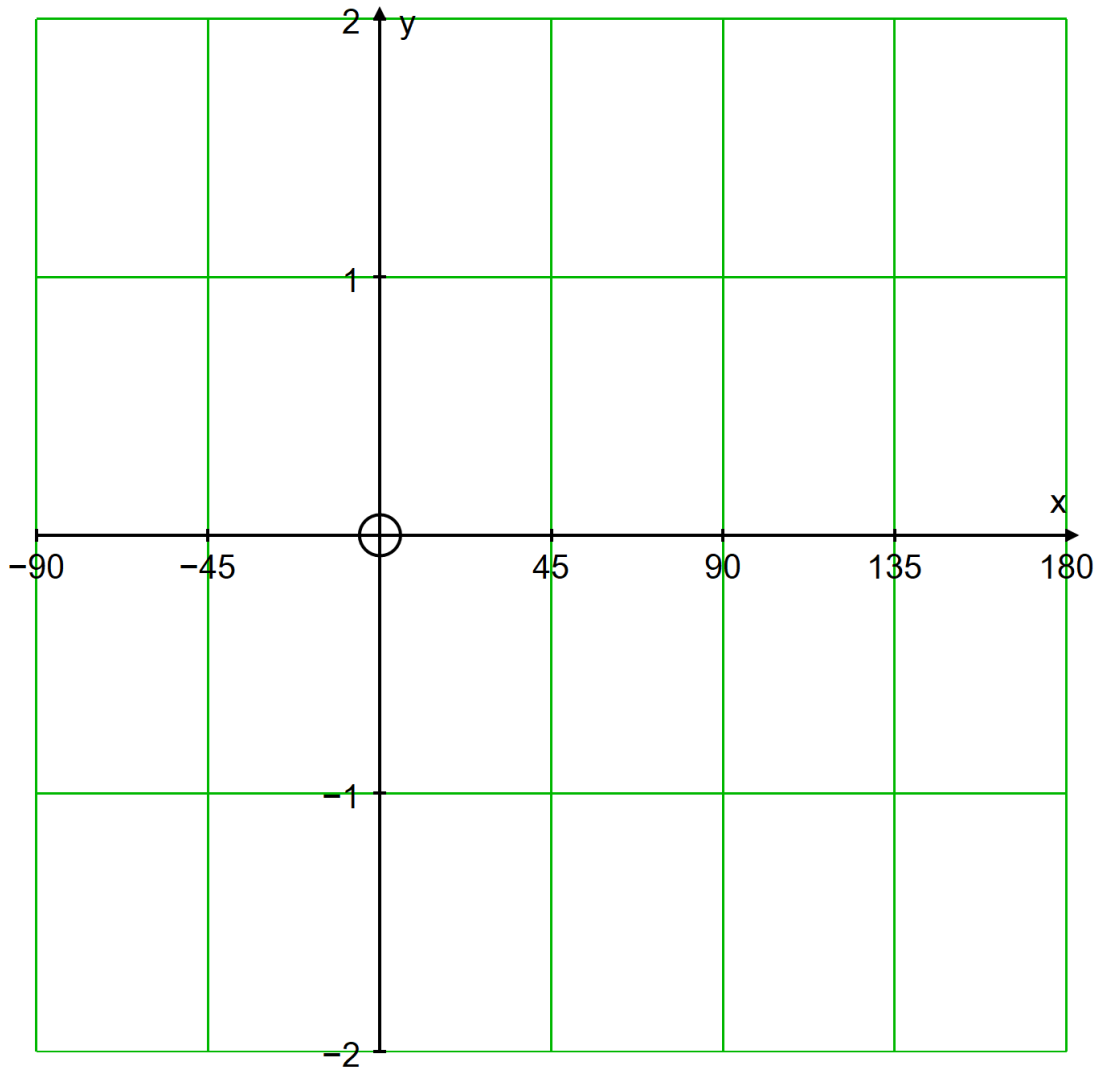- K lies on HI with $KL \parallel HJ$.
- $HJ = 29$ units.

H

29

K

J

9

L

12

I

Determine the length of IK.

(5)
[11]

QUESTION 6

Given: $f(x) = \sin 2x$ and $g(x) = \cos(x + 45°)$ for $x \in [-90°; 180°]$.

(a)   Sketch the graphs of f and g on the same set of axes below. **Label all intercepts with axes, turning points and end points.**

(6)

(b)   Determine the values of $x \in [-90°; 180°]$ for which $f(x) = g(x)$.
        Show all calculations.

(5)

(c)   Use your graph to determine the value(s) of $x \in [-90°; 180°]$ for which:

        (1) $f(x) \geq g(x)$

(1)

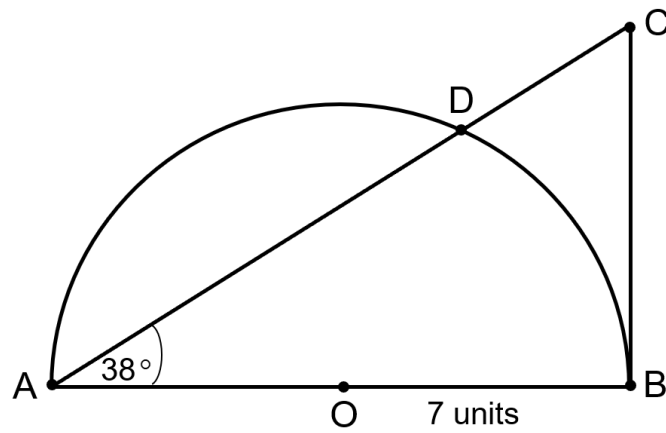   (2) $f(x).g(x) < 0$

(2)
[14]

73 marks

SECTION B

QUESTION 7

In the diagram below:

- ABD is a semi-circle with centre O.
- CB is a tangent to the semi-circle at B.
- OB = 7 units and $C\hat{A}B = 38°$

Determine the length of CD, correct to one decimal place.
State all relevant reasons with your statements.

[7]

QUESTION 8

Given: $\cos\theta = \dfrac{a^2 - b^2}{a^2 + b^2}$ where $0 < b < a$ and $\sin\theta < 0$.

(a)   Determine $\tan\theta$ in terms of a and b. **Show all working.**

(4)

(b)   If it is further given that $a = 3$ and $b = 2$, solve for θ, where $\theta \in (0°; 720°)$.

(3)
[7]

QUESTION 9

(a)   Simplify the following into an expression containing only one trigonometric ratio of θ:

$$\frac{\frac{1}{2}\cos(90°+\theta)-\sin\theta.\sin(\theta-90°)}{\cos^2(180°-\theta)-2\cos(-\theta)+\cos^2(\theta+90°)}$$

(7)

(b)   (1)   Prove the following identity:

$$\frac{\sin\theta.\tan\theta}{\tan 2\theta.\left(1-\tan^2\theta\right)} = \frac{\sin\theta}{2}$$
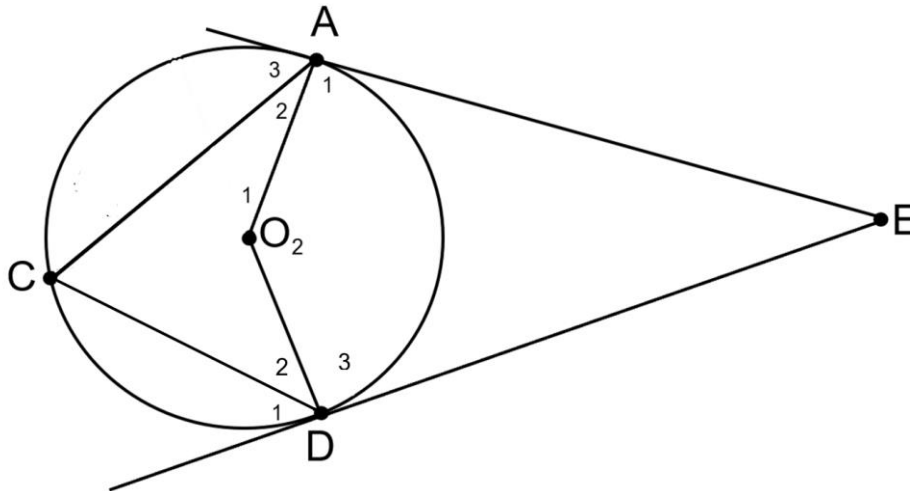
(5)

(2)   Determine the general solution for the values of θ for which the identity is **not valid**. Write your answer as ONE single statement.

(5)
[17]

QUESTION 10

In the diagram:

- A, C and D lie on the circle with centre O.
- AE and DE are tangents to the circle at A and D respectively.
- $A\hat{C}D = 68°$

State all relevant reasons with your statements.
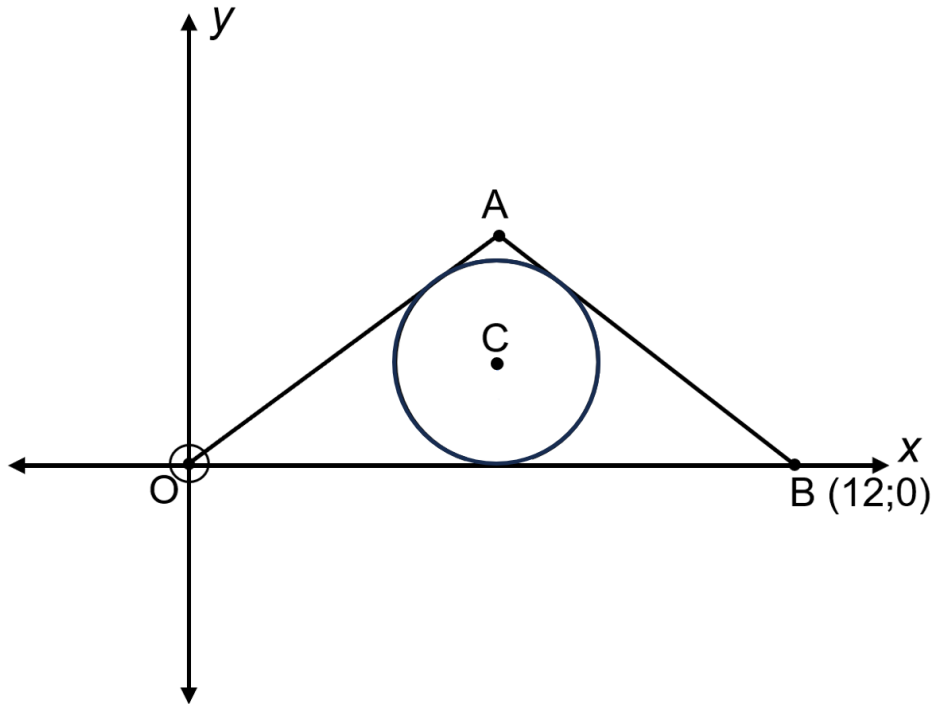
(a)   Prove that AODE is a cyclic quadrilateral.

(4)

(b)   Determine: $\hat{E}$

(4)
[8]

QUESTION 11

In the diagram below:

- The equation of the circle with centre C is: $x^2 + y^2 - 12x - 4y + p = 0$.
- AO, OB and AB are tangents to the circle.
- B is the point (12;0).

(a)   Determine the value of *p.*

(5)

(b)   Determine the equation of AB. Show all your working.

(7)

(c)   A second circle with centre (0;9) and with point (2;3) that lies on the circumference is drawn on the same set of axes. Do the two circles intersect? Show all working.

(5)
[17]

QUESTION 12

In the diagram:

- Circle with centre O has diameter AOC and chords AB and BC.
- Chords AD and EC intersect at F with OF perpendicular to EC.

- AOC bisects $B\hat{C}E$.
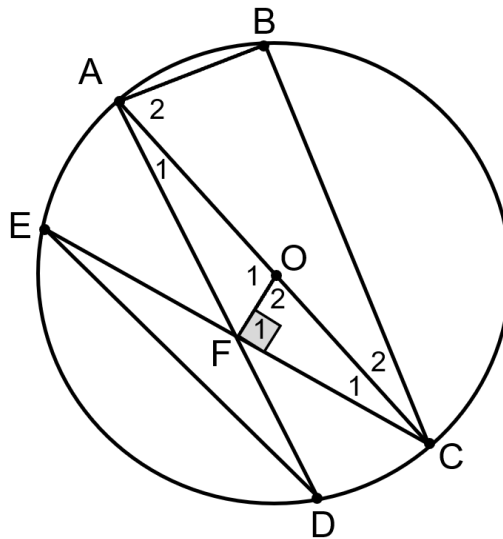
State all relevant reasons with your statements.

(a)   Prove the $\triangle ABC \;///\; \triangle OFC$

(4)

(b)   Determine the numerical value of BC : FC

(2)

(c)   Hence prove that: $\left(\dfrac{AC}{2}\right)^2 - \left(\dfrac{AB}{2}\right)^2 = DF \times FA$
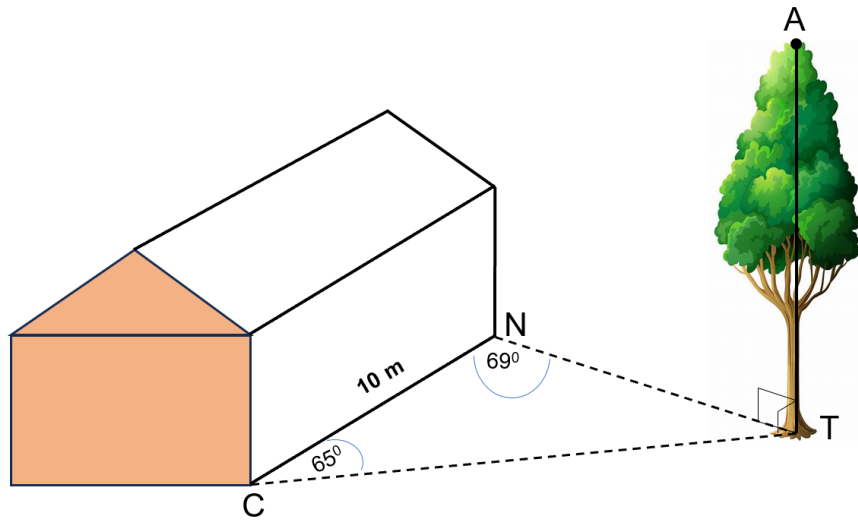
(7)
[13]

QUESTION 13

In the diagram below, a house is in the shape of a prism with a triangular shaped roof with one side CN = 10m.
A vertical tree is growing at a point T in the garden.

$N\hat{C}T = 65°$ and $C\hat{N}T = 69°$.
The angle of elevation of the top of the tree from C is $43,5°$.

Is it possible for the tree to hit the house if it falls? Show all working.

[8]

77 marks

Total: 150 marks

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.